Price Impact

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Broad Lesson Plan

Motivation

- Implicit assumption in asset pricing: zero friction
- [Çetin et al., 2004] extend the fundamental theorem of asset pricing to liquidity risk in an economy where the security price depends on the order size.
- Intra-day trading strategies: must buy (sell) immediately upon a buy (sell) signal
- Position closing
- Hedging (spread trading, option etc)
- Agency trade execution
- BIG QUESTION: If you trade x contracts with a market order, how much do you pay for the price impact of your trade?

Basic Economics

- Liquidity providers are on the sell side, and consumers are on the buy side.
- **What is Price Impact?**
  The compensation the liquidity providers receive for rendering this service is simply the market impact cost paid by the liquidity demanders.
  Barra's Market Impact Model Handbook (1997)
- Fundamental assumption of market microstructure: There is an **unobservable frictionless price** S_t and an observable transaction price P_t.
  - [Roll, 1984]
  - Huang and Stoll; Madhavan, Richardson, and Roomans (1997)
  - Glosten-Harris (1988)
  - Tee and Ting (2017)

Existing Price Impact Functions

- Kyle (1985): Linear function of order flow
- Bloomberg and Barra's market impact models: square roof of order size V

$$\frac{1}{2}\frac{\text{bid-ask spread}}{\text{price}} + \sqrt{\frac{\sigma^2/3}{250}}\sqrt{\frac{V}{0.3 \times \text{EDV}}}$$

- JP Morgan's model

$$95\% \times \frac{1.4}{\text{EPV} \times \sqrt{\text{EDV}}}\sigma^2 V^{3/2} + 5\% \times \frac{0.187}{\sqrt{\text{EDV}}}\sigma^2\sqrt{V}$$

EDV: the expected daily volume EPV expected period volume.

- R^2 is less than 10%.

Order Flow

- Order flow is transaction volume that is signed [Lyons (2001)].

- Liquidity providers: market makers and traders who submit limit orders

- Liquidity consumers: traders who need immediacy in transactions
  - ★ Outright market order
  - ★ Marketable limit order

- Liquidity consumers generate order flow X_t over a time period:
  - ★ More buying volume than selling volume: $X_t > 0$
  - ★ More selling volume than buying volume: $X_t < 0$

Is Order Flow Observable?

- With tick-by-tick data: YES.

Data of Small Nikkei 16H Futures from Bloomberg

Date	Time	Type	Price	Contracts	Sign	Signed Volume
2016-02-08	9:45:55	B	16755	210		
2016-02-08	9:45:55	A	16760	144		
2016-02-08	9:45:55	T	16760	1	1	1
2016-02-08	9:45:55	A	16760	143		
2016-02-08	9:45:55	T	16755	1	-1	-1
2016-02-08	9:45:55	A	16760	150		
2016-02-08	9:45:56	B	16755	209		
2016-02-08	9:45:56	A	16760	149		
2016-02-08	9:45:56	B	16755	219		
2016-02-08	9:45:56	A	16760	149		
2016-02-08	9:45:56	B	16755	213		
2016-02-08	9:45:56	A	16760	149		
2016-02-08	9:45:56	T	16760	3	1	3
2016-02-08	9:45:56	T	16755	2	-1	-2
2016-02-08	9:45:56	A	16760	146		
2016-02-08	9:45:56	B	16755	211		
2016-02-08	9:45:56	A	16760	146		
2016-02-08	9:45:56	B	16755	212		
2016-02-08	9:45:56	T	16760	2	1	2
	2-second		order flow	=	1	3

Model Setup

- Observable trade (or transaction) price P_t
- Observable trade size X_t
  - ★ positive for buy and negative for sell at time t
- Unobservable frictionless price S_t
  - ★ price used by classical asset pricing theories
- Assumption of trade price

$$P(t, X_t) = S_t \xi(X_t) \tag{1}$$

- Assumptions of impact function $\xi(X_t) = \exp\big(f(X_t)\big)$
  1. $\xi(0) = 1$, i.e., $f(0) = 0$
  2. $f(x)$ is non-decreasing, twice differentiable
- Intuition behind: $X_t \gtrless 0 \implies P(t, X_t) \gtrless S_t$

Assumptions on Stochastic Processes

- Under the physical measure $\mathbb{P}$, we introduce two standard Brownian motions w_t and z_t with correlation

$$\mathrm{d}w_t\,\mathrm{d}z_t = \rho\,\mathrm{d}t$$

- Order flow (positive or negative) X_t is a mean-reverting process

$$\mathrm{d}X_t = c(m - X_t)\,\mathrm{d}t + \eta\,\mathrm{d}w_t \tag{2}$$

  m: long-run average, c: speed of mean reversion, η: volatility of X_t

- Frictionless price S_t is a geometric Brownian motion

$$\frac{\mathrm{d}S_t}{S_t} = \mu_S\,\mathrm{d}t + \sigma_S\,\mathrm{d}z_t \tag{3}$$

- More general models are possible but...

Transitory versus Permanent

- Because of the correlation, $\mathrm{d}w_t \mathrm{d}z_t = \rho\,\mathrm{d}t$, it follows that

$$\frac{\mathrm{d}S_t}{S_t}\mathrm{d}X_t = \rho\eta\sigma_S \mathrm{d}t. \tag{4}$$

- $S_t := S(t)$ does not depend on the order flow directly, but it is related to the order flow volatility η.
- $\rho = 0 \quad \Longrightarrow \quad$ pure temporary effect of order flow
- Can be interpreted as some sort of indirect “permanent” price impact of order flow on the true price

Log-Return Process for Trade Price

- From (1),

$$\mathrm{d}\log P_t = \mathrm{d}\log S_t + \mathrm{d}f(X_t)$$

- Applying Itô's rule, we obtain

$$\mathrm{d}\log P_t = \left[\mu_S - \frac{1}{2}\sigma_S^2 + c(m - X_t)g(X_t) + \frac{1}{2}\eta^2 g'(X_t)\right]\mathrm{d}t + \sigma_S\,\mathrm{d}z_t + \eta\,g(X_t)\,\mathrm{d}w_t \tag{5}$$

where $g(x) := f'(x) \geq 0$, which will be a parameter

- Let $g(0) = \ell > 0$, which be a parameter of the model.

Simple Return Process for Trade Price

- In view of the stochastic terms in (5), define

$$\sigma_P^2(x) := \sigma_S^2 + \eta^2 g^2(x) + 2\rho\eta\sigma_S g(x) \tag{6}$$

- It follows that

$$\frac{\mathrm{d}P_t}{P_t} = \mu_P(X_t)\mathrm{d}t + \sigma_S \mathrm{d}z_t + \eta g(X_t)\mathrm{d}w_t, \tag{7}$$

where

$$\begin{aligned}\mu_P(x) = \mu_S + \{c(m-x) + \rho\eta\sigma_S\}g(x) \\ + \frac{1}{2}\eta^2\{g'(x) + g^2(x)\}.\end{aligned} \tag{8}$$

Market Prices of Risks

- Let $\lambda_t^z := \lambda^z(X_t)$ and $\lambda_t^w := \lambda^w(X_t)$ denote the market prices of risks arising from z_t and w_t, respectively.
- Define

$$\mathrm{d}\widetilde{z}_t := \mathrm{d}z_t + \lambda_t^z \mathrm{d}t, \quad \mathrm{d}\widetilde{w}_t := \mathrm{d}w_t + \lambda_t^w \mathrm{d}t$$

- Let $\mathbb{Q}$ be a probability measure that makes the processes $\widetilde{z}_t$ and $\widetilde{w}_t$ standard Brownian motions with $\mathrm{d}\widetilde{z}_t \mathrm{d}\widetilde{w}_t = \rho \mathrm{d}t$.
- Substituting these into (7), we obtain

$$\frac{\mathrm{d}P_t}{P_t} = \big(\mu_P(X_t) - \sigma_S \lambda^z(X_t) - \eta g(X_t)\lambda^w(X_t)\big)\mathrm{d}t + \sigma_S \mathrm{d}\widetilde{z}_t + \eta g(X_t)\mathrm{d}\widetilde{w}_t \qquad (9)$$

under $\mathbb{Q}$.

No Arbitrage Opportunity

- Suppose that price manipulation is not possible for P_t.
- In this case, we can assume that there is no arbitrage opportunity in the market.
- Under $\mathbb{Q}$, the denominated trade price P_t/B_t is a martingale, where $B_t = \mathrm{e}^{rt}$ and r denotes the risk-free spot rate.
- It follows from (9) that

$$r = \mu_P(x) - \sigma_S \lambda^z(x) - \eta g(x) \lambda^w(x) \tag{10}$$

- Assume that investors demand a premium $\kappa(x)$ for exposing themselves to price impact risk.
- Accordingly, we have

$$\lambda^z(x) = \frac{\mu_S - r + \kappa(x)}{\sigma_S}, \quad \kappa(x) \geq 0 \tag{11}$$

Ordinary Differentiable Equation

- Combining (10) and (11), and in view of (8), we obtain the ODE satisfied by $g(x)$, i.e.,

$$0 = -\kappa(x) + \{c(m - x) + \rho\eta\sigma_S - \eta\lambda^w(x)\}g(x) + \frac{1}{2}\eta^2\{g'(x) + g^2(x)\} \tag{12}$$

- Note that the ODE (12) involves the market price of risk $\lambda^w(x)$ associated with the order flow X_t.
- Need to specify $\lambda^w(x)$ in order to solve the ODE explicitly.

Bernoulli Differentiable Equation

- Define

$$s(x) := 2\frac{\kappa(x)}{\eta^2}$$

- Define a function of order flow:

$$p(x) := \frac{2}{\eta^2}\{c(m-x) + \rho\eta\sigma_S - \eta\lambda^w(x)\} \tag{13}$$

- It follows from (12) and (13) that

$$0 = -s(x) + g'(x) + p(x)g(x) + g^2(x) \tag{14}$$

which is the inhomogeneous Bernoulli equation.

The Price Impact Model

- Given the solution $g(x)$ of (14) at least numerically, we have the following price impact models:
- Under the physical measure $\mathbb{P}$,

$$\frac{\mathrm{d}P_t}{P_t} = \big(\mu_S + \kappa(x) + \eta g(X_t)\lambda^w(X_t)\big)\mathrm{d}t + \sigma_S \mathrm{d}z_t + \eta g(X_t)\mathrm{d}w_t,$$
$$\mathrm{d}X_t = c(m - X_t)\mathrm{d}t + \eta \mathrm{d}w_t; \qquad (15)$$

- Under the risk-neutral measure $\mathbb{Q}$,

$$\frac{\mathrm{d}P_t}{P_t} = r\,\mathrm{d}t + \sigma_S \mathrm{d}\widetilde{z}_t + \eta g(X_t)\mathrm{d}\widetilde{w}_t,$$
$$\mathrm{d}X_t = \{c(m - X_t) - \eta\lambda^w(X_t)\}\mathrm{d}t + \eta \mathrm{d}\widetilde{w}_t, \qquad (16)$$

where $\mathrm{d}z_t \mathrm{d}w_t = \mathrm{d}\widetilde{z}_t \mathrm{d}\widetilde{w}_t = \rho \mathrm{d}t$.

A General Solution

- Consider the ODE (14), and let $\alpha(x)$ be a special solution, i.e.,

$$\alpha'(x) + p(x)\alpha(x) + \alpha^2(x) = s(x) \tag{17}$$

- Define, for $\beta(x) = p(x) + 2\alpha(x)$,

$$\phi(x) = \mathrm{e}^{-\int_0^x \beta(y)\mathrm{d}y}, \quad \Phi(x) = \int_0^x \phi(y)\mathrm{d}y \tag{18}$$

1 A general solution to the ODE (14) is given by

$$g(x) = \alpha(x) + \frac{A\phi(x)}{A\Phi(x) + B}, \qquad \beta(x) = p(x) + 2\alpha(x),$$

where A and B are integration constants.

Proof

- From (18), we have $\phi'(x) = -\beta(x)\phi(x)$.
- Denoting $\Gamma(x) := A\Phi(x) + B$, it follows that

$$g'(x) = \alpha'(x) - \frac{A\beta(x)\phi(x)\Gamma(x) + A^2\phi^2(x)}{\Gamma^2(x)}$$

- Also, we have

$$p(x)g(x) = p(x)\alpha(x) + \frac{Ap(x)\phi(x)\Gamma(x)}{\Gamma^2(x)}$$

$$g^2(x) = \alpha^2(x) + \frac{2A\alpha(x)\phi(x)\Gamma(x)}{\Gamma^2(x)} + \frac{A^2\phi^2(x)}{\Gamma^2(x)}$$

- Summing these terms, we obtain

$$\alpha'(x) + p(x)\alpha(x) + \alpha^2(x) + \frac{A\phi(x)\{-\beta(x) + p(x) + 2\alpha(x)\}}{\Gamma(x)}$$

which is equal to $s(x)$.

Solution to the Bernoulli Equation

- The boundary conditions: $g(0) = \ell$ and $f(0) = 0$.
- Denote $\alpha := \alpha(0)$, and from the proposition, we have

$$\ell = g(0) = \alpha + \frac{A}{B}$$

- Moreover, $f(x) = \int_0^x \alpha(y)\mathrm{d}y + \log[A\Phi(x) + B]$, which implies $0 = \log B$, i.e., $B = 1$
- Thus, the price impact function is given by

$$f(x) = \int_0^x \alpha(y)\mathrm{d}y + \log[(\ell - \alpha)\Phi(x) + 1] \tag{19}$$

where $\Phi(x)$ is defined in (18), i.e., $\beta(x) = p(x) + 2\alpha(x)$ and

$$\phi(x) = \mathrm{e}^{-\int_0^x \beta(y)\mathrm{d}y}, \quad \Phi(x) = \int_0^x \phi(y)\mathrm{d}y.$$

Special Cases

- Difficult to obtain special solution $\alpha(x)$ in general.
- Two special cases to obtain solutions in closed form.
  1. $p(x) = p$ and $s(x) = s$: Special solution $\alpha(x)$ is a constant, i.e., $\alpha(x) = \alpha$.
  2. $s(x) = 0$: We have the trivial special solution, i.e., $\alpha(x) = 0$.
- For Case 1, we have $p\alpha + \alpha^2 = s$ from (17).
- The solution is given by

$$\alpha = \frac{-p + \sqrt{p^2 + 4s}}{2}, \quad s \geqq 0 \tag{20}$$

- Hence, $\beta(x) = \beta$ is also a constant and given by

$$\beta = p + 2\alpha = \sqrt{p^2 + 4s} > 0 \tag{21}$$

Special Case 1

- In this case, we have

$$\phi(x) = \mathrm{e}^{-\beta x}, \quad \Phi(x) = \frac{1}{\beta}(1 - \mathrm{e}^{-\beta x}),$$

where α and β are given in (20) and (21), respectively.

- From (19), the impact function in this case is given by

$$f(x) = \alpha x + \log\left[\frac{\ell - \alpha}{\beta}(1 - \mathrm{e}^{-\beta x}) + 1\right] \tag{22}$$

which is a sum of a linear impact function and a concave function.

Linear Functional Form as a Special Case

- For the logarithm in (22) to be well defined, the applicability of this price impact function is restricted to

$$x > -\frac{1}{\beta} \log\left(1 + \frac{\beta}{\ell - \alpha}\right).$$

- Therefore, for (22) to be applicable to all x, we have to set $\ell = \alpha$.
- Consequently, the nonlinear term in (22) vanishes and the price impact function reduces to $f(x) = \alpha x$.
- Consistent with the theoretical finding of linearity by [Kyle, 1985]

Special Case 2

- In this case, we have $\alpha(x) = 0$ so that $\beta(x) = p(x)$.
- From (19), the impact function is given by

$$f(x) = \log[1 + \ell\Phi(x)], \quad g(x) = \frac{\ell\phi(x)}{1 + \ell\Phi(x)}, \quad x \in I, \tag{23}$$

where

$$\phi(x) = e^{-\int_0^x p(y)\mathrm{d}y}, \quad \Phi(x) = \int_0^x \phi(y)\mathrm{d}y.$$

- Here, I denotes the interval on which the function $f(x)$ is well defined, i.e., $1 + \ell\Phi(x) > 0$ for $x \in I$.

S–Shaped Impact Function

- Empirical studies suggest that the impact function is S-shaped.
- The simplest model (Case 1) is not appropriate.
- We need more complicated models for this purpose with appropriate restrictions.

2 Suppose that $p(x)$ is increasing in x and it has a unique solution $x^* \in I$ for $p(x) = 0$. Then, the impact function $f(x)$ is convex for $x < x^*$ and concave for $x > x^*$.

Proof

- From (18), since $\phi'(x) = -p(x)\phi(x)$, it is readily seen that

$$g'(x) = -\ell\phi(x)\frac{p(x)[1+\ell\Phi(x)] + \ell\phi(x)}{[1+\ell\Phi(x)]^2}$$

- Let $h(x) := p(x)[1+\ell\Phi(x)] + \ell\phi(x)$, we have

$$h'(x) = p'(x)[1+\ell\Phi(x)]$$

- Since $1+\ell\Phi(x) > 0$ for $x \in I$, $h(x)$ is increasing, $h(x) < 0$ for $x < x^*$ and $h(x) > 0$ for $x > x^*$.
- Since $\ell\phi(x) > 0$, we have $g'(x) \geq 0$ for $x < x^*$ and $g'(x) \leq 0$ for $x > x^*$ under the condition.

A Simple S–Shaped Model

- The simplest model that guarantees the S-shaped impact function is to assume $\kappa(x) = 0$ and $p(x) = p + qx$ with $q > 0$.
- We then have

$$\phi(x) = e^{-px - \frac{q}{2}x^2}, \quad \Phi(x) = \int_0^x e^{-py - \frac{q}{2}y^2} dy \tag{24}$$

- Since $x^* = -p/q$, the impact function $f(x)$ is convex for $x < -p/q$ and it is concave for $x > -p/q$, where $x \in I$.
- What is the interval I for which $1 + \ell\Phi(x) > 0$?
- To this end, denoting $N(x) := \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$, we obtain

$$\Phi(x) = \frac{\sqrt{2\pi} e^{p^2/2q}}{\sqrt{q}} \left(N(\sqrt{q}(x + p/q)) - N(p/\sqrt{q}) \right) \tag{25}$$

which is well defined for all $x > 0$.

A Simple S–Shaped Model, Cont.

- On the other hand, for $x < 0$, denoting $\hat{x} = -x > 0$, we have

$$\Phi(-\hat{x}) = -\frac{\sqrt{2\pi}e^{p^2/2q}}{\sqrt{q}}\left(N(p/\sqrt{q}) - N(\sqrt{q}(-\hat{x} + p/q))\right) \quad (26)$$

- Since $N(x)$ is monotonically increasing, we conclude that if

$$1 - \ell\sqrt{\frac{2\pi}{q}}e^{p^2/2q}N(p/\sqrt{q}) > 0 \quad (27)$$

  then the impact function $f(x)$ is well defined for all x.

- In the following, we assume that the parameters p and q satisfy the condition (27) for given $\ell > 0$, where $p(x) = p + qx$ with $q > 0$.

Some Simplification

- For the later purpose of empirical studies, we further simplify as

$$\eta\lambda^w(x) = -\hat{c}x, \quad \hat{c} > c > 0$$

- By the definition (13), we then have

$$p(x) = \frac{2}{\eta^2}[cm + \rho\eta\sigma_S] + \frac{2}{\eta^2}(\hat{c} - c)x$$

which implies that

$$p = \frac{2}{\eta^2}[cm + \rho\eta\sigma_S], \quad q = \frac{2}{\eta^2}(\hat{c} - c) > 0 \tag{28}$$

- From (15), we have under $\mathbb{P}$

$$\frac{\mathrm{d}P_t \mathrm{d}X_t}{P_t} = \eta\{\sigma_S\rho + \eta g(X_t)\}\mathrm{d}t \tag{29}$$

- Hence, our model has a stochastic correlation (also SV).

Order Flow Dependent Effective Spread

- The assumption is rewritten as

$$\log P_t = \log S_t + f(X_t).$$

- Since it involves log prices, the price impact

$$f(X_t) = \log P_t - \log S_t$$

  is in percent (%) or basis points (0.01%).

- In the market microstructure literature, the price differential $2 \times |P_t - S_t|$ is known as the **effective spread**. which does not depend on trading volume.

- Analogously, we can interpret $f(X_t) = \log P_t - \log S_t$ as positive half effective spread in percent for buy if $f(X_t) > 0$, and for sell if $f(X_t) < 0$, which are dependent on X_t.

Econometric Specification

- The assumption leads to

$$\mathrm{d}\log P_t = \mathrm{d}\log S_t + \mathrm{d}f(X_t).$$

- We obtain

$$\begin{aligned} r_t := \log P_t - \log P_{t-1} &= \log S_t - \log S_{t-1} + f(X_t) - f(X_{t-1}) \\ &= \left(\mu_S - \frac{1}{2}\sigma_S^2\right)\Delta t + f(X_t) - f(X_{t-1}) + \epsilon_t, \end{aligned}$$

where $\Delta t = 1$ is set at 1-minute sampling period, and ϵ_t is the noise term.

- In this way, we obtain an econometric specification:

$$r_t = a + f(x_t) - f(x_{t-1}) + \epsilon_t. \tag{30}$$

Constraints from Theory

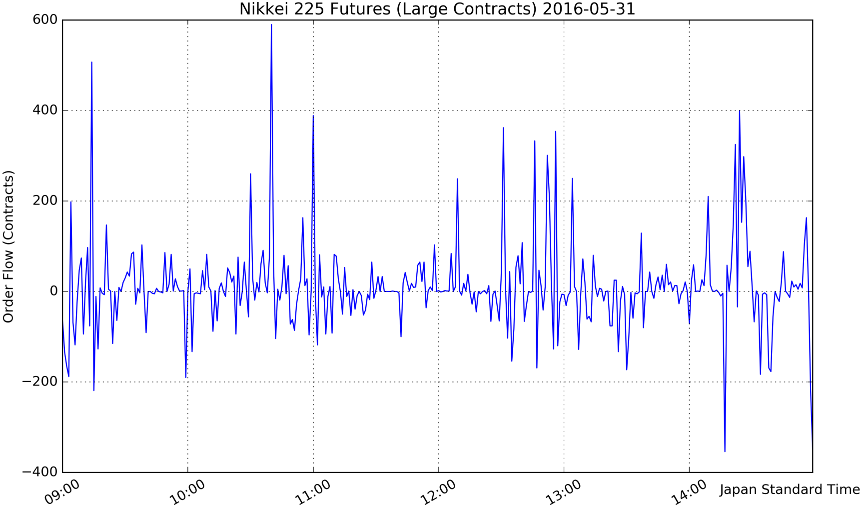
- $\ell := g(0) > 0$
- Given that $p(x) = p + qx$, for it to be non-trivial and economically sensible, $q > 0$.
- Note that p is not constrained to be positive.
- $\ell \sqrt{\frac{2\pi}{q}} \mathrm{e}^{\frac{p^2}{2q}} N\left(p/\sqrt{q}\right) < 1$

Five Futures Specifications on Nikkei 225 Index

Name of Futures	Bloomberg Ticker	Contract Size	Tick Size (index points)	Number of Trades	Contracts Traded
JPX Big Nikkei	NK	¥1,000	10	9,148	42,607
JPX Mini Nikkei	NO	¥100	5	51,587	432,331
SGX Nikkei	NI	¥500	5	27,001	49,704
CME Nikkei	NH	¥500	5	5,258	10,312
CME Nikkei	NX	$50	5	3,870	6,309

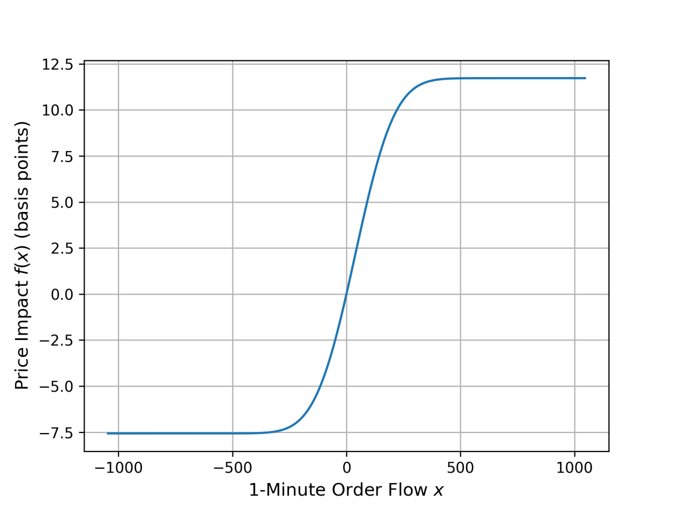
- Regular trading hours: 9 AM to 3 PM Japan Standard Time (JST)
- Notional amount in ¥or $ = contract size $\times$ index points
  - ★ One tick is ¥10,000 for NK; ¥500 for NO; ¥2,500 for NI and NH, and $25 for NX.

An Example: 1-Minute Order Flow

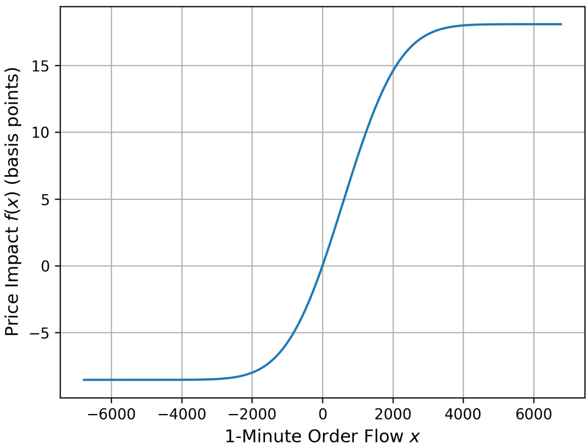


Data source: Bloomberg tick data

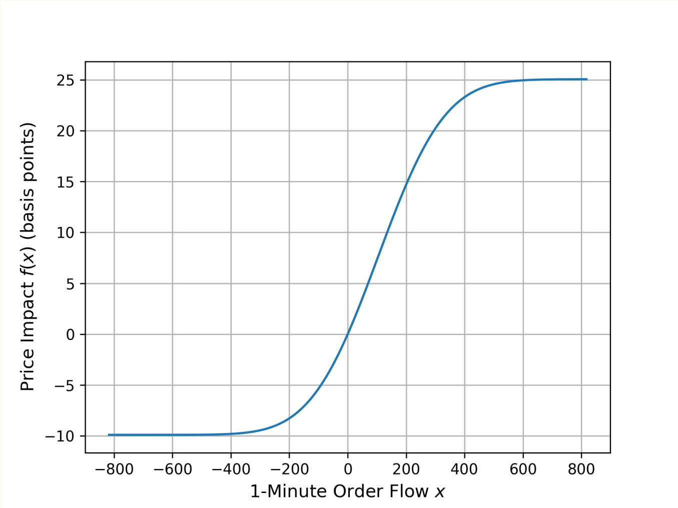
An Estimation Result of JPX's NK



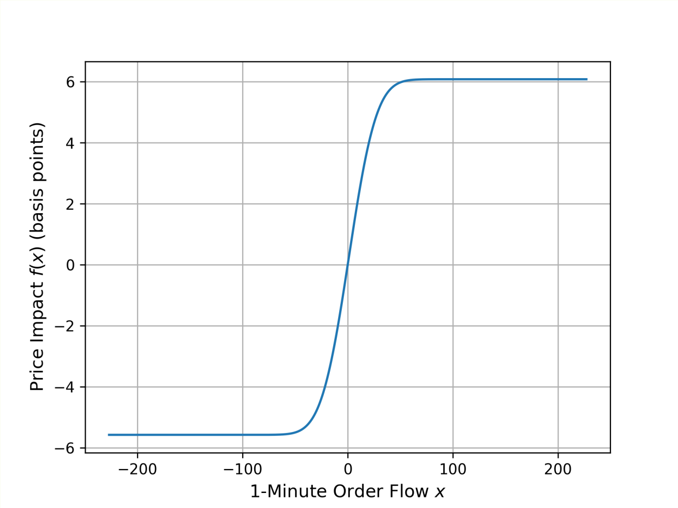
An Estimation Result of JPX's NO



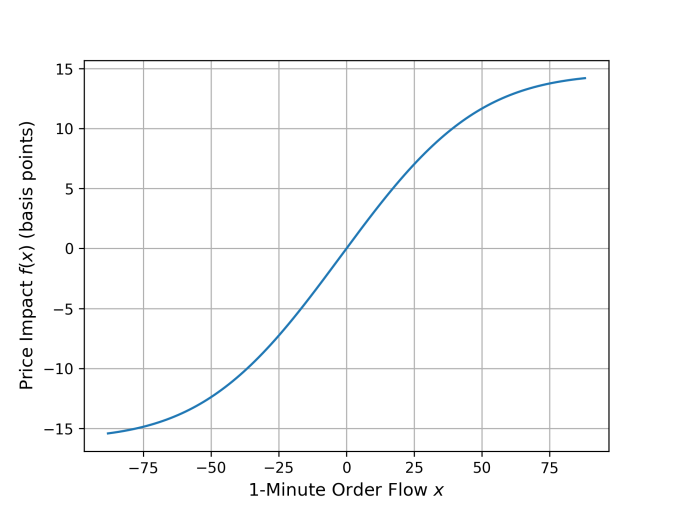
An Estimation Result of SGX's NI



An Estimation Result of CME's NH



An Estimation Result of CME's NX



Market Price of Liquidity Risk

- Market price of liquidity risk per contract: $\dfrac{c_\star}{\eta}$

- It is the liquidity-risk adjusted premium paid by liquidity consumers to liquidity providers.

- We use the maximum likelihood method to estimate c and m.

- We compute $\hat{\eta}$ as the unbiased sample variance.

- Given the estimates $\hat{\eta}$, $\hat{c}$, and $\hat{q}$, the market price of liquidity risk is computed as a **liquidity ratio**:

$$\widehat{\lambda^w}(-1) := \frac{\frac{1}{2}\hat{\eta}^2\hat{q} + \hat{c}}{\hat{\eta}}. \tag{31}$$

Estimation Results

Estimates	NK	NO	NI	NH	NX
$\widehat{\ell}$ (bps)	0.126	0.0139	0.122	1.03	1.38
	(0.0278)	(0.00251)	(0.0359)	(0.386)	(0.524)
$\widehat{p}$	-3.25	-0.17	-5.05	-80.71	-103.97
	(2.00)	(0.126)	(3.83)	(48.9)	(747)
$\widehat{q}$	75.85	0.36	181.32	8,850	30,665
	(38.5)	(0.204)	(110.63)	(5,750)	(22,042)
$\widehat{a}$	2.31	2.01	2.26	2.02	2.04
	(3.33)	(2.74)	(2.97)	'(3.03)	(3.03)
$\widehat{R}^2$ (%)	27.82	39.40	23.15	18.42	16.81
Lower bound (bps)	13.82	23.4	8.65	7.73	6.53
Upper bound (bps)	25.17	37.21	15.79	31.82	17.11
$\widehat{c}$	2.72	2.29	5.11	2.53	3.51
$\widehat{m}$	6.8	41.27	-1.69	0.17	-0.46
$\widehat{\eta}$	91.7	662.46	86.96	21.95	10.26
Liquidity risk per contract (%)	3.32	0.36	6.67	21.24	49.98

Conclusions

- An S-shape price impact function of 1-minute order-flow from first principle
- Dependent on the volatility of the frictionless price and also that of the volatility of order flow non-linearly
- Liquidity ratio: a gauge of order-flow risk adjusted premium that market makers (submitters of limit orders) require
- The parameters p, q, ℓ can be forecasted and employed for pre-trade analysis.

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